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Generalization of Pairwise Fuzzy Semi Continuous Map in Fuzzy Bitopological Spaces

Abstract

In this paper we have introduced and studied generalized pairwise fuzzy semi continuous map. We have obtained equivalent conditions for a map from one fuzzy bitopological space to another to be generalized pairwise fuzzy semi continuous map, Also we have established some significant properties of generalized pairwise fuzzy semi continuous map and constructed some examples.

Keywords: Fuzzy Topological Space, Fuzzy Regular Open Set, Fuzzy Regular Closed Set, Fuzzy Bitopological Space, (i,j)-Fuzzy Semi Open Set.

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Introduction

The concept of fuzzy sets was introduced and defined by Zadeh [7] in 1965. As fuzzy set is very useful concept to define vagueness so many mathematicians have worked on it. In 1968 Chang [2] has introduced the concept of fuzzy topological space as a generalization of topological space. The concept of fuzzy semi open(fuzzy semi closed) set and fuzzy regular open(fuzzy regular closed) set was introduced by Azad [1] in 1981. Thakur and Malviya [5] have introduced the notion of (i,j)-fuzzy semi open set and (i,j)-fuzzy semi closed set in 1996.

Aim of the Study

In this paper we have generalized the concept of pairwise fuzzy semi continuous map in fuzzy bitopological spaces.

Preliminaries

Let X be a non-empty universal crisp set. A **fuzzy set** μ on the set X is a mapping $\mu: X \rightarrow I$, where $I = [0,1]$. For $x \in X$, the real number $\mu(x)$ in [0,1] is called **grade of membership** of x in the fuzzy set μ. A fuzzy point x_p, $x \epsilon X$ is a fuzzy set on X defined as

$$x_p(y) = \begin{cases} p \ (p \in (0,1]) & \text{for } y = x, \\ 0 & \text{otherwise} \end{cases}$$

for all $y \in X$.

Definition 2.1 [2]

Let τ be a collection of fuzzy sets on X. Then τ is said to be a Fuzzy Topology on X if it satisfies following conditions:

(i) The fuzzy sets 0 and 1 are in τ, where 0,1:X→ I, are defined as 0(x)=0 and 1(x)=1, for all $x \in X$.

(ii) If $\alpha, \beta \in \tau$ then $\alpha \cap \beta \in \tau$.

(iii) If $\{\alpha_j\}_{j \in J}$, J is an index set, is any family of fuzzy sets on X and $\alpha_j \in \tau$, for all $j \in J$ then $\bigcup_{j \in J} \alpha_j \in \tau$.

The pair (X,τ) is called ***fuzzy topological space****.*

Definition 2.2 [2]

Let (X,τ) be a fuzzy topological space. The members of the collection τ are called **fuzzy open sets** and complement of fuzzy open sets are called **fuzzy closed sets**. For a fuzzy set α on X the **closure** of α is denoted by Cl(α) and is defined to be the intersection of all fuzzy closed sets in X containing α. The **interior** of α is denoted by Int(α), and is defined to be the union of all fuzzy open sets in X which are contained in α.

Definition 2.3 [1]

Let (X,τ) be a fuzzy topological space. A fuzzy set λ on X is called **fuzzy regular open set** if $Int(Cl(\lambda)) = \lambda$ and is called **fuzzy regular closed set** if $Cl(Int(\lambda)) = \lambda$.

Remark

Each fuzzy regular open (regular closed) set is fuzzy open (closed) set. The converse may not be true.

Definition 2.4 [4]

Let X be a non empty crisp set and let τ_1 and τ_2 be fuzzy topologies on X. Then the triplet (X,τ_1,τ_2) is called a **fuzzy bitopological space**. The members of τ_i $(i = 1,2)$ are called **τ_i -fuzzy open sets** and their complement are called **τ_i –fuzzy closed sets**.

Definition 2.5 [5]: A fuzzy set λ on a fuzzy bitopological space (X,τ_1,τ_2) is called **(i,j)-fuzzy semi-open set** if there exists a τ_i-fuzzy open set ϑ such that $\vartheta \le \lambda \le \tau_j - Cl(\vartheta)$ and it is called **(i,j)-fuzzy semi-closed set** if there exists a τ_i-fuzzy closed set μ such that $\tau_j - int(\mu) \le \lambda \le \mu$.

Definition 2.6 [2]: A mapping $f:(X,\tau)\rightarrow (X^*,\tau^*)$ is said to be **fuzzy continuous map** if the inverse image of every fuzzy open fuzzy set in X^* is fuzzy open set in X.

Definition 2.7 [5]: A mapping $f:(X,\tau_1,\tau_2)\rightarrow (X^*,{\tau^*}_1,{\tau^*}_2)$ is said to be **pairwise fuzzy continuous** if $f:(X,\tau_1)\rightarrow (X^*,{\tau^*}_1)$ and $f:(X ,\tau_2)\rightarrow (X^*,{\tau^*}_2)$ are fuzzy continuous maps.

Definition 2.8 [6]: A mapping $f:(X,\tau_1,\tau_2)\rightarrow (X^*,{\tau^*}_1,{\tau^*}_2)$ is said to be **pairwise fuzzy irresolute** if the inverse image of every (i,j)-fuzzy semi-open set in X^* is (i,j)-fuzzy semi-open set in X.

Generalized Pairwise Fuzzy Semi-Continuous Map

In this section we have studied generalized pairwise fuzzy semi contiuous map. We have obtained equivalent conditions for a map $f:(X,\tau_1,\tau_2)\rightarrow (X^*,{\tau^*}_1,{\tau^*}_2)$ to be generalized pairwise fuzzy semi-continuous map.

Definition 3.1 [5]

A mapping $f:(X,\tau_1,\tau_2)\rightarrow (X^*,{\tau^*}_1,{\tau^*}_2)$ is said to be pairwise fuzzy semi continuous if the inverse image of every ${\tau^*}_i$-fuzzy open set in X^* is (i,j)-fuzzy semi-open set in X.

Remark

Every pairwise fuzzy continuous mapping is pairwise fuzzy semi-continuous. But the converse may not be true.

Definition 3.2

A mapping $f:(X,\tau_1,\tau_2)\rightarrow (X^*,{\tau^*}_1,{\tau^*}_2)$ is said to be generalized pairwise fuzzy semi continuous if the inverse image of every ${\tau^*}_i$-fuzzy regular open set in X^* is (i,j)-fuzzy semi-open set in X.

Proposition 3.3

If $f:(X,\tau_1,\tau_2)\rightarrow (X^*,{\tau^*}_1,{\tau^*}_2)$ is pairwise fuzzy semi-continuous map then it is generalized pairwise fuzzy semi semi-continuous map.

Proof

Let $f: X \rightarrow X^*$ be a pairwise fuzzy semi-continuous map where (X,τ_1,τ_2) and $(X^*,{\tau^*}_1,{\tau^*}_2)$ are fuzzy bitopological spaces. Let α be a ${\tau^*}_i -$ fuzzy regular open set in X^*. Then α is a fuzzy $\tau_i -$ open in X^*. Since $f: X \rightarrow X^*$ is pairwise fuzzy semi continuous, $f^{-1}(\alpha)$ is (i,j)-fuzzy semi-open set in X. Hence we find that inverse image of each ${\tau^*}_i -$ fuzzy regular open set in X^* is (i,j)-fuzzy semi-open set in X. Therefore $f: X \rightarrow X^*$ is generalized pairwise fuzzy semi-continuous map.

However a generalized pairwise fuzzy semi-continuous map may not be pairwise fuzzy semi continuous. We have following example:

Example 3.4

Let $X = \{x, y, z\}, \ and \ X^* = \{a, b\}$. Let α, β $be\, fuzzy\ sets\ on\ X$ $and\ \gamma, \delta$ $be fuzzy\ sets\ on\ X^*$, defined as

$\alpha(x) = 0.3, \alpha(y) = 0.4, \alpha(z) = 0.4;$

$\beta(x) = 0.4, \beta(y) = 0.5, \ \beta(z) = 0.4;$

$\gamma(a) = 0.5, \gamma(b) = 0.4$ and $\delta(a) = 0.4, \delta(b) = 0.3.$

Let $\tau_1 = \{0, \alpha, \beta, 1\}$, $\tau_2 = \{0,1\}$, ${\tau^*}_1 = \{0, \gamma, \delta, 1\}$ and ${\tau^*}_2 = \{0,1\}$. Then (X,τ_1,τ_2) and $(X^*,{\tau^*}_1,{\tau^*}_2)$ are fuzzy bitopological spaces. The mapping $f:(X,\tau_1,\tau_2) \rightarrow (X^*,{\tau^*}_1,{\tau^*}_2)$ defined as $f(x) = a$, $f(y) = f(z) = b$, is generalized pairwise fuzzy semi-continuous map but not pairwise fuzzy semi continuous map.

Corollary 3.5

If $f:(X,\tau_1,\tau_2)\rightarrow (X^*,{\tau^*}_1,{\tau^*}_2)$ is pairwise fuzzy irresolute map then it is generalized pairwise fuzzy semi-continuous map.

We have obtained equivalent conditions for a map $f:(X,\tau_1,\tau_2)\rightarrow (X^*,{\tau^*}_1,{\tau^*}_2)$ to be generalized pairwise fuzzy semi continuous map.

Theorem 3.6

Let $(X,\tau_1,\tau_2)\ and\ (X^*,{\tau^*}_1,{\tau^*}_2)$ be fuzzy bitopological spaces and let $f: X \rightarrow X^*$, be a mapping. Then the following conditions are equivalent:

(i) f is generalizesd pairwise fuzzy semi-continuous map ;

(ii) for every ${\tau^*}_i$ - fuzzy regular closed set B in X^*, $f^{-1}(B)$ is (i,j)-fuzzy semi-closed set in X;

(iii) for every fuzzy point x_α of X and every ${\tau^*}_i$ - fuzzy regular open set M such that $f(x_\alpha)\epsilon M$ there is a (i,j) –fuzzy semi-open set A in X such that $x_\alpha \in A$ and $f(A) \le$M.

Proof

(i) $\Rightarrow$ (ii): let B be a ${\tau^*}_i$ - fuzzy regular closed set in X^* . Then $B^c = 1 - B$ is a ${\tau^*}_i$-fuzzy regular open set in X^*. Since f is generalized pairwise fuzzy semi continuous map, $f^{-1}(B^c)$ is (i,j)-fuzzy semi-open set in X. This implies 1- $f^{-1}(B^c) = 1 - f^{-1}(1 - B)$=$f^{-1}(B)$ is (i,j)-fuzzy semi-closed set in X.

(ii) $\Rightarrow$ (i): Let U be a ${\tau^*}_i$ - fuzzy regular open set in X^*. Then $U^c = 1 - U$ is a ${\tau^*}_i$-fuzzy regular closed set in X^*. Now by given condition (ii), $f^{-1}(1 - U) = 1 - f^{-1}(U)$ is (i, j) –fuzzy semi-closed set in X. This implies $f^{-1}(U)$ is (i,j)-fuzzy semi-open set in X.

(i) $\Rightarrow$ (iii): Let $f: (X, \tau_1, \tau_2) \rightarrow (X^*, {\tau^*}_1, {\tau^*}_2)$ be a generalized pairwise fuzzy semi-continuous map. Let M be a ${\tau^*}_i$ - fuzzy regular open set in X^* and let x_α be a fuzzy point in X such that $f(x_\alpha) \epsilon M$. Since f is generalized pairwise fuzzy semi-continuous map, $f^{-1}(M)$ is (i,j) –fuzzy semi-open set in X, $x_\alpha \epsilon$ $f^{-1}(M)$, and $f(f^{-1}(M)) \le M$.

$(iii) \Rightarrow (i)$: *Let V be a ${\tau^*}_i$ - fuzzy regular open set in X^*. Let x_α be a fuzzy point of X such that $f(x_\alpha)\epsilon V$. Then by given condition there is a (i,j)-fuzzy semi-open set A in X such that $x_\alpha \epsilon A$ and $f(A) \le V$. This implies $A \le f^{-1}(V)$. Hence we find that $f^{-1}(V)$ is the union of (i,j) –fuzzy semi-open sets in X. Since arbitrary union of (i,j) –fuzzy semi-open set is (i,j) –fuzzy semi-open set, it follows that*

$f^{-1}(V)$ is (i,j)-fuzzy semi-open in X . Thus $f: X \to X^*$ is a generalized pairwise fuzzy semi continuous map.

Theorem 3.7

Let (X,τ_1,τ_2), (Y,δ_1,δ_2), $(Z,\vartheta_1,\vartheta_2)$ be fuzzy bitopological spaces. If $f: X \to Y$ is pairwise fuzzy irresolute map and $g: Y \to Z$ is generalized pairwise fuzzy semi-continuous map then $g \circ f: X \to Z$ is generalized pairwise fuzzy semi-continuous map.

Proof

let μ be $\tau^*{}_i$ - fuzzy regular open set in Z. Since $g: Y \to Z$ is generalized pairwise fuzzy semi-continuous map, $g^{-1}(\mu)$ is (i,j)- fuzzy semi open set in Y. Again since $f: X \to Y$ is pairwise fuzzy irresolute map so $f^{-1}(g^{-1}(\mu))$ is (i,j) –fuzzy semi-open set in X. This implies $(g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$ is (i,j) –fuzzy semi-open set in X. Hence $g \circ f: X \to Z$ is generalized pairwise fuzzy semi-continuous map.

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